Identification with External Instruments in Structural VARs under Partial Invertibility

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Abstract

This paper discusses conditions for identification of structural shocks with external instruments in VARs, under partial invertibility. This is a very general condition, often of empirical relevance, and less stringent than the standard full invertibility, sometimes required for SVARs and LPs with controls. We show that in this case dynamic responses can be recovered provided that a condition of limited lead-lag exogeneity of the instrument holds. This condition allows the instrument to be contaminated by leads and lags other invertible shocks. We evaluate our results in a simulated environment and provide an empirical application to the case of monetary policy shocks.

Keywords: Identification with External Instruments; Structural VAR; Invertibility; Monetary Policy Shocks.
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1 Introduction

A central endeavour in empirical macroeconomics is the study of the dynamic causal effects that structural shocks have on macroeconomic variables. Since Sims (1980), this has been typically accomplished by employing Structural VARs (SVARs). An almost always maintained assumption in the SVAR literature is that of 'fundamentalness', or 'invertibility' of the structural shocks, given the chosen model. If this assumption holds, all the structural shocks can be recovered from the current and lagged values of the observables included in the VAR. Under invertibility, the VAR innovations are a linear combination of the structural shocks and, given the variance-covariance matrix of the residuals, the causal relationships are identified up to an orthogonal matrix that defines the contemporaneous relationships. A lot of creativity in the SVAR literature has been devoted to the formulation of appropriate identifying assumptions to inform the choice of this orthogonal matrix. The structural moving average, obtained by inverting the identified SVAR, allows inference on the dynamic causal effects of the structural shocks, represented in the form of impulse response functions (IRFs).

In contrast with standard statistical identifications, an important advancement in the more recent practice has seen the adoption of external instruments for the identification of structural shocks.[1] These instruments – that can be thought of as noisy observations of the shocks of interest –, can be used either in conjunction with Structural VARs (SVAR-IV, also called Proxy-VARs), or with direct regression methods, such as Jordà (2005)'s Local Projections (LP-IV with or without controls). The assumption of invertibility, however, is still required both in SVAR-IV and LP-IV with controls for the system to be fully identified (see discussion in Stock and

[1]This rapidly expanding research programme, surveyed in Ramey (2016), has produced, among other applications, a number of instruments for the identification of monetary policy (e.g. Romer and Romer, 2004; Gürkaynak et al., 2005; Gertler and Karadi, 2015; Miranda-Agrippino and Ricco, 2017; Paul, 2017), fiscal spending (e.g. Ramey, 2011; Ricco et al., 2016; Ramey and Zubairy, 2018), tax (e.g. Romer and Romer, 2010; Leeper et al., 2013; Mertens and Ravn, 2012), government asset purchases (Fieldhouse and Mertens, 2017; Fieldhouse et al., 2018), oil (e.g. Hamilton, 2003; Kilian, 2008), and technology news shocks (e.g. Miranda-Agrippino et al., 2018).

Watson, 2018).[2]

This paper discusses the conditions for identification with external instruments in Structural VARs under the assumption of partial invertibility of the shock of interest. This is the empirically relevant case in which the researcher is only interested in 'partially' identifying the system, that is, in estimating the dynamic effects of just one (or a subset) of the structural shocks that can be assumed to be recoverable from the VAR residuals.

We show that, in general, fairly weak conditions are required to achieve identification. In particular, under partial invertibility, other than the standard relevance and contemporaneous exogeneity conditions, the instrument has to fulfil a limited lead-lag exogeneity condition. This ensures that the VAR innovations and the instrument are related only via the structural shock of interest. Hence, the instrument can be contaminated by leads and lags of other partially invertible shocks, while still achieving correct identification. Our results allow to extend the application of SVAR-IV (and LP-IV with controls) methods to the many empirically relevant cases in which the shock of interest is arguably invertible, while some of the other structural disturbances may be non-invertible.[3]

We make three contributions. First, we show that under partial invertibility a covariance-stationary stochastic vector process admits a 'semi-structural' representation that is the sum of two terms, orthogonal to one another. The first one only depends on the current realisations of the partially invertible shocks. The second instead combines leads and lags of the remaining non-invertible shocks. This result

[2]Stock and Watson (2018) observe that direct methods, such as local projections, do not need to explicitly assume invertibility of the system under strict exogeneity of the instrument at all leads and lags. However, if lagged observables are required as control variables for an instrument that violates the lead-lag exogeneity condition, then, in general, the same invertibility conditions of a structural VAR are required.

[3]Our results are a generalisation of the special case in which the shock of interest is observed without error (i.e. the instrument is the shock), discussed in Stock and Watson (2018). In this case, they note, the assumption of invertibility can be dispensed with for the validity of SVAR-IV. Intuitively, if the instrument perfectly reveals the shock, the dynamic causal effects can be consistently estimated by a distributed lag regression of the variables of interest on the observed shock.

implies that if the VAR lag order correctly captures the autocorrelation structure of the Wold representation, the impulse response functions obtained from the partially identified structural moving average are the dynamic causal effects of the shock of interest.

Second, we show that under partial invertibility SVAR-IV methods (and LP-IV with controls) achieve identification under much weaker conditions on the external instrument than LP-IV methods without controls. Intuitively, for partially invertible SVAR-IVs, it is enough to assume that the instrument correlates with the VAR residuals only via the shock of interest. Hence, the semi-structural representation implies that the instrument can be contaminated by leads or lags (but not contemporaneous realisations) of any of the other partial-invertible shocks in the system. We call this requirement a limited lead-lag exogeneity condition.

Third, we discuss the empirically likely case in which the VAR is misspecified along some dimensions – e.g. inappropriate lag order, missing moving average components, missing variables, and missing higher order terms –, and hence fails to correctly capture the data generating process. While in this case the dynamic responses will generally be biased, if one can still assume that the shock of interest is partially invertible, the impact effects are correctly identified. This provides a simple way to gauge the contamination of an instrument versus the misspecification of the chosen model. If one can assume partial invertibility across different specifications of an empirical model, an instrument that fulfils the conditions for identification delivers stable impact responses but unstable IRFs across models. In this case, increasing the number of lags and/or using a larger information set should help stabilising the dynamics responses by providing a better approximation of the Wold representation. Conversely, an instrument that violates the lead-lag exogeneity condition is likely to deliver unstable impact responses across different models.

We provide an application of our results using artificial data from a stylised standard New-Keynesian DSGE model with price stickiness and four shocks – monetary policy, government spending, technology, and an inflation-specific shock. The system is by construction partially invertible in the monetary policy – i.e. the residuals of the Taylor rule. However, due to the introduction of technology news (see e.g. Beaudry and Portier, 2006; Barsky and Sims, 2011), and fiscal foresight (see Ramey, 2011; Leeper et al., 2013), a VAR in output growth, inflation, government spending and the policy interest rate fails the 'poor man's invertibility condition' of Fernandez-Villaverde et al. (2007), and is hence unable to recover all the four shocks. We use this simulated environment to study the identification of monetary policy shocks with external instruments. Our results validate our discussion. Under partial invertibility, an instrument contaminated by leads or lags of an invertible shocks correctly recover impacts and dynamic responses to the shock of interest, provided that the VAR is correctly specified. If, instead, the instrument is contaminated by a non-invertible shock, the degree of distortion in the estimated IRFs depends on how pervasive the shock is, that is, on how much of the variance in the system it accounts for.

Lastly, we provide an empirical application of our results by examining popular instruments for the identification of monetary policy shocks in a monthly VAR for US data. We consider three variants of the high-frequency instruments popularised by Gürkaynak et al. (2005) to identify monetary policy shocks. We show that two of these are likely to fail the limited lead-lag exogeneity condition, and hence they recover impact responses of output and prices that are strongly dependent on the model specification and composition. The third instrument, constructed as in Miranda-Agrippino and Ricco (2017) with a pre-whitening step to remove correlation with other shocks, recovers impact responses that are invariant to the VAR specification.

This paper builds and expands on the econometric literature supporting the use of IV in macroeconomics. The SVAR-IV techniques were first introduced by Stock (2008), and then explored in Stock and Watson (2012) and Mertens and Ravn (2013). The use of instrumental variables for identification in direct regressions (LP-IV), with or without controls, has been proposed independently by Jordà et al. (2015) and

Ramey and Zubairy (2018). The econometric conditions for instruments validity in the direct regression without control variables have first appeared in lecture notes by Mertens (2014). Stock and Watson (2018) have recently provided a unified discussion of the use of external instruments in macroeconomics, discussed the conditions for instruments validity with control variables and relation to full invertibility, and explored the connections between the SVAR-IV and LP-IV methods.[4]

This paper is close in spirit to Forni et al. (2018) – which expands on the approach proposed by Giannone and Reichlin (2006) and results in Forni and Gambetti (2014) –, and studies the conditions under which a SVAR is informative enough to estimate the dynamic effects of a shock. While we share the emphasis on partial invertibility (referred to in Forni et al. 2018 as informational sufficiency), our paper focuses on the recent debate on the use of IV in empirical macro, and on its interaction with misspecifications in the modelling choices.

The paper is organised as follows. In Section 2 we review the concepts of invertibility and fundamentalness and some other useful results in the literature; a reader familiar with these concepts can skip the section. Sections 3 an 4 collect our main results. Here we discuss partial identification, and how this allows for semi-structural representations of covariance-stationary vector processes, and lay out the conditions for the identification of structural shocks in SVAR-IV under partial invertibility of the shock of interest. In Section 5 we analyse the case of misspecified systems. We apply the concept discussed in this paper using artificial data from a NK-DSGE in Sections 6 and an empirical application in Section 7. Finally, Section 8 concludes.

2 Non-Fundamental Representations

To introduce the concept of non-fundamentalness, let us consider a covariance-stationary $n \times 1$ vector stochastic process Y_t, for $t \in \mathbb{Z}$, with rational spectral density and be-

[4]Plagborg-Møller and Wolf (2018) discuss the connection between IRF estimated with VARs and LP methods.

longing to a Hilbert space $L^2(\Omega, \mathcal{F}, P)$ for some probability space $(\Omega, \mathcal{F}, P)$.[5] We define the Hilbert space generated by all the observations of Y_t up to time t as $\mathcal{H}_t^Y = \overline{\text{span}}\{Y_{t-j}, j \geq 0\}$. The process Y_t is a linear process and a VARMA(p,q) if it is a stationary solution of the stochastic difference equation

$$\Phi(L)Y_t = \Psi(L)u_t \qquad u_t \sim \mathcal{WN}(0, \Sigma_u) \ , \tag{1}$$

where $\Phi(L)$ and $\Psi(L)$ are generic autoregressive (AR) and moving average (MA) filters of order p and q respectively

$$\Phi(L) = \sum_{i=0}^{p} \Phi_i L^i \ , \qquad \Psi(L) = \sum_{i=0}^{q} \Psi_i L^i, \tag{2}$$

and u_t are the stochastic disturbances of the data generating process (i.e. the 'structural shocks' in the economic jargon), generally assumed to be orthogonal or orthonormal processes. If the process is causal – i.e., $det(\Phi(L))$ has all roots outside the unit circle, $det(\Phi(z)) \neq 0 \ \forall z = \zeta_i$ such that $|\zeta_i| < 1$ –, then it can be written as a (possibly infinite) MA in the structural shocks u_t

$$Y_t = \Theta(L)u_t, \qquad u_t \sim \mathcal{WN}(0, \Sigma_u). \tag{3}$$

Definition 1. (Invertibility and Fundamentalness) *Let Y_t be defined as in Eq.* (1)*, and with structural MA representation as in Eq.* (3)*.*

(i) If $det(\Psi(z))$ – and hence $det(\Theta(z))$ – has all roots outside the unit circle, i.e.

$$det(\Theta(z)) \neq 0, \qquad \forall z = \zeta_i \ s.t. \ |\zeta_i| < 1, \tag{4}$$

[5] In the economic literature, the issue of non-fundamentalness (see Rozanov, 1967; Hannan, 1970) was first pointed out by Hansen and Sargent (1980, 1991) in a purely theoretical setting, while Lippi and Reichlin (1993, 1994) provided the first empirical application. Other more recent contributions on fundamentalness in macro models are in Chari et al. (2004), Christiano et al. (2007) and Fernandez-Villaverde et al. (2007). A useful review is in Alessi et al. (2011).

then the process in Eq. (1) *is said to be invertible, and* u_t *are said to be* Y_t*-fundamental (i.e.* $\mathcal{H}_t^Y = \mathcal{H}_t^u$ *and the stochastic disturbances can be recovered from current and past realisation of the process* Y_t*).* Y_t *can be written in VAR form as*

$$A(L)Y_t = \Theta_0 u_t \ , \tag{5}$$

where Θ_0 *is an n-dimensional matrix.*

(ii) If $det(\Theta(z))$ *has at least one root inside the unit circle, then the process in Eq.* (1) *is 'non-invertible', and* u_t *is said to be* Y_t*–non-fundamental (i.e.* $\mathcal{H}_t^Y \subset \mathcal{H}_t^u$*).*

(iii) If $det(\Theta(L))$ *has at least one root on the unit circle, the process is said to be non-invertibile, but* u_t *are* Y_t*-fundamental (*$\mathcal{H}_t^Y = \mathcal{H}_t^u$*).*

The Wold Representation Theorem guarantees that Y_t always admits a Wold decomposition of the form

$$Y_t = \eta_t + C(L)\nu_t \qquad \nu_t \sim \mathcal{WN}(0, \Sigma_\nu), \tag{6}$$

where $C(L) = \sum_j C_j L^j$ is a causal (i.e. no terms with $C_j \neq 0$ for $j < 0$), time-independent, square summable filter with $C_0 = \mathbb{I}_n$ and η_t is a deterministic term that, without loss of generality, we will disregard in the following in order to focus on purely non-deterministic processes. ν_t is the Wold innovation process – an uncorrelated sequence – to Y_t

$$\nu_t = Y_t - Proj(Y_t|Y_{t-1}, Y_{t-2}, \dots) \ , \tag{7}$$

that, by definition, belongs to the space generated by present and past values of Y_t (i.e. $\mathcal{H}_t^\nu = \mathcal{H}_t^Y$, since we are assuming Y_t to be a purely non-deterministic process). Given the invertibility of $C(L)$, we can rewrite Eq. (6) in a VAR form

$$A(L)Y_t = \nu_t \qquad A_0 = \mathbb{I}_n \ . \tag{8}$$

If the Wold representation has absolute summable coefficients, then it admits a VAR representation with coefficient matrices that decay to zero rapidly; hence, it can be well approximated by a finite order VAR process. This is always the case for causal finite-order ARMA processes.

If the structural shocks u_t are Y_t–fundamental, then u_t and ν_t generate the same space ($\mathcal{H}_t^u = \mathcal{H}_t^\nu, \ \forall t$). This implies that

$$\nu_t = \Theta_0 u_t \ , \tag{9}$$

where Θ_0 is non-singular. Hence, the structural disturbances u_t can be determined from current and lagged values of Y_t

$$u_t = Proj(u_t | Y_t, Y_{t-1}, \dots) \ . \tag{10}$$

If, however, the process is not invertible, and u_t is not Y_t–fundamental, the space generated by the VAR innovations does not coincide with that spanned by the structural shocks, i.e. $\mathcal{H}_t^\nu \subset \mathcal{H}_t^u$. The following result guarantees that the Wold and the structural MA representations (Eq. 3) are connected by a class of transformations generated by means of Blaschke matrices.

Theorem 1. *Let Y_t be a covariance-stationary vector process with rational spectral density, i.e. an ARMA process. Let $Y_t = C(L)\nu_t$ be a fundamental representation of Y_t, i.e.*

(i) ν_t is a white noise vector;

(ii) C(L) is a matrix of rational functions in L with no poles of modulus smaller or equal to unity (Causality);

(iii) $det(C(L))$ has no roots of modulus smaller than unity (Invertibility).

Let $Y_t = \Theta(L)u_t$ be any other MA representation, i.e. one which fulfils (i), and (ii), *but not necessarily (iii). Then*

$$C(L) = \Theta(L)B(L) \ ,$$

where $B(L)$ is a Blaschke matrix.

Blaschke matrices are filters capable to flip the roots of a fundamental representation inside the unit circle (see Lippi and Reichlin, 1994). A complex-valued matrix $B(z)$ is a Blaschke matrix if: (i) It has no poles inside the unit circle; (ii) $B(z)^{-1} = B^{*'}(z^{-1})$, where * indicates the complex conjugation.[6] The following result guarantees that any Blaschke matrix can be written as the product of orthogonal matrices, and matrices with a Blaschke factor as one of their entries.

Theorem 2. *Let $B(z)$ be an $n \times n$ Blaschke matrix, then $\exists\, m \in \mathbb{N}$ and $\exists\, \zeta_i \in \mathbb{C}$ for $i = 1, \ldots, m$ such that*

$$B(z) = \prod_{i=1}^{m} K(\zeta_i, L)R_i \ , \tag{11}$$

where R_i are orthogonal matrices, i.e. $R_i R_i^{'} = \mathbb{I}_n$, and

$$K(\zeta_i, L) = \begin{pmatrix} \mathbb{I}_{n-1} & 0 \\ 0 & \dfrac{z - \zeta_i}{1 - \zeta_i^* z} \end{pmatrix} \ , \tag{12}$$

are matrices with a Blaschke factor as one of the entries.

The above results indicate that in general we can connect the structural and the Wold representation using a Blaschke matrix $B(L)$, that is

$$Y_t = \Theta(L)u_t = \Theta(L)B(L)^{-1}B(L)u_t = C(L)\nu_t, \tag{13}$$

where $B(L)$ flips the roots of the Wold fundamental representation inside the unit

[6] See Lippi and Reichlin (1994) for a proof of Theorems 1 and 2.

circle to obtain the structural MA. Hence,

$$\nu_t = \Theta_0 B(L) u_t \ . \tag{14}$$

In the case in which the structural representation is invertible, $B(L)$ is just the product of the orthogonal matrices R_i.[7]

It is important to observe that, as it is clear from Eqs. (11-12), Blaschke factors may be acting only on a subset of the shocks. The remaining shocks can be recovered from current and past realisations of the variables, and are said to be partially invertible. We discuss this relevant case in the next section.

3 Partial Invertibility

The property of invertibility guarantees identifiability of all the structural disturbances of a correctly specified VAR. In such a case, the problem of identification amounts to finding the correct matrix Θ_0 that connects the VAR residuals to the structural shocks as in Eq. (9). However, phenomena such as anticipation and foresight of economic shocks, which are often a feature of rational expectation models, can generate non-invertible representations (see e.g. Leeper et al., 2013). In such cases, the search for the correct Blaschke matrix can be a daunting problem (see Lippi and Reichlin, 1994).

In most empirical applications, however, often only a subset of the structural innovations is of interest. For example, one may want to identify only a monetary policy shock, or an oil price shock. This is the case of 'partial identification', when only a subset of the column entries of the matrix polynomial $\Theta_0 B(L)$ that maps the Wold residuals into the structural shocks is of interest. In such a setting, the relevant condition is that of partial invertibility of the subset of the shocks of interest.

[7] A matrix R_i is an orthogonal matrix if its transpose is equal to its inverse, i.e. $R_i' R_i = R_i R_i' = \mathbb{I}_n$. The group $O(n)$ of the $n \times n$ orthogonal matrices is spanned by $n(n-1)/2$ unrestricted parameters.

Definition 2. (Partial Invertibility) *Let Y_t be a covariance-stationary $n \times 1$ vector stochastic process, with rational spectral density, solution to the ARMA equation $\Phi(L)Y_t = \Psi(L)u_t$, where u_t is a $n \times 1$ vector of stochastic disturbances (structural shocks) $u_t \sim \mathcal{WN}(0, \Sigma_u)$. Y_t admits a Wold representation of the form $Y_t = C(L)\nu_t$ for a vector of innovations $\nu_t \sim \mathcal{WN}(0, \Sigma_\nu)$. A structural shock $u_t^i \in u_t$ is invertible and Y_t–fundamental if*

$$u_t^i = Proj(u_t^i | Y_t, Y_{t-1}, \dots) \ . \tag{15}$$

Hence, u_t^i is a linear combination of the innovations ν_t, that is, there exist an n-dimensional unit norm vector λ, with $\lambda\lambda' = 1$, such that

$$\kappa u_t^i = \lambda' \nu_t \ , \tag{16}$$

where κ is a constant of proportionality.

Under partial invertibility, a straightforward application of Theorem 2 guarantees that Eq. (14) reads

$$\nu_t = \Theta_0 B(L) u_t = \widetilde{B}(L) u_t = [\tilde{b}_1 \ \ \tilde{b}_2(L)] u_t \ , \tag{17}$$

where $\tilde{b}_1$ is a $n \times m$ matrix, and $\tilde{b}_2(L)$ is a matrix polynomial of dimensions $n \times (n-m)$ obtained as a combination of Blaschke factors and orthogonal transformations, where m is the number of partially invertible shocks.

Proposition 1. (Semi-structural Moving Average Representation) *Let the covariance stationary vector process Y_t be a solution to*

$$\Phi(L)Y_t = \Psi(L)u_t \qquad u_t \sim \mathcal{WN}(0, \Sigma_u) \ , \tag{18}$$

and let $\Psi(L)$ be a non-invertible moving average filter, i.e. $det(\Psi(z)) = 0$ for some

ζ_i such that $|\zeta_i| < 1$. Let the Wold representation of Y_t be equal to

$$Y_t = C(L)\nu_t \qquad \nu_t \sim \mathcal{WN}(0, \Sigma_\nu). \tag{19}$$

If the system is partially invertible in a shock u_t^i for some $i \in n$, viz. exists a unit-norm vector λ such that $\lambda'\nu = \kappa u_t^i$, then Y_t admits a semi-structural moving average representation of the form

$$Y_t = \kappa C(L)\lambda u_t^i + C(L)\tilde{\lambda}\xi_t \ , \tag{20}$$

where $\tilde{\lambda}$ is such that $\tilde{\lambda}'\lambda = 0_{(n-1)\times 1}$, $\tilde{\lambda}'\tilde{\lambda} = \mathbb{I}_{n-1}$ and $\mathbb{E}(u_t^i\xi_t') = 0$.

Proof. Let us consider a non singular matrix

$$\Lambda' = \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} \tag{21}$$

such that $\tilde{\lambda}'\lambda = 0_{(n-1)\times 1}$, and $\tilde{\lambda}'\tilde{\lambda} = \mathbb{I}_{n-1}$.[8] Λ is an orthogonal matrix, $\Lambda'\Lambda = \Lambda\Lambda' = \mathbb{I}_n$.[9] Also,

$$\Lambda'\nu_t = \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} \nu_t \ = \begin{pmatrix} \kappa u_t^i \\ \xi_t \end{pmatrix}. \tag{22}$$

[8]It is possible to constructively obtain a non-singular matrix Λ by observing that since λ is normalised to be of unitary norm, it can be thought of as the first column of an orthogonal matrix. $\tilde{\lambda}$ has to live in the orthogonal complement subspace of $\mathbb{R}^n$ of the space defined by λ. This space is spanned by a generic basis of $n-1$ independent vectors of norm one, orthogonal to λ. Any such a base can be used as column vectors of $\tilde{\lambda}$. Λ is then non-singular, and an orthogonal matrix.

[9]This follows trivially from the assumptions on the sub-matrices λ and $\tilde{\lambda}$ and the choice of a non-singular Λ. First, observe that

$$\Lambda'\Lambda = \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix} = \begin{pmatrix} \lambda'\lambda & \lambda'\tilde{\lambda} \\ \tilde{\lambda}'\lambda & \tilde{\lambda}'\tilde{\lambda} \end{pmatrix} = \begin{pmatrix} 1 & 0_{1\times(n-1)} \\ 0_{(n-1)\times 1} & \mathbb{I}_{n-1} \end{pmatrix} = \mathbb{I}_n.$$

This also implies

$$\mathbb{I}_n = \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix} \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix}^{-1} \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix}^{-1} \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} = \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix} \left(\begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix} \right)^{-1} \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} = \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix} \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} = \Lambda\Lambda'.$$

$\xi_t \equiv \tilde{\lambda}'\nu_t$ is a combination of structural shocks, and is orthogonal to u_t^i at different lags and leads, i.e. $\xi_t' u_t^i = (\tilde{\lambda}'\nu_t)' u_t^i \propto (\tilde{\lambda}'\nu_t)'\lambda'\nu_t = \nu_t'\tilde{\lambda}\lambda'\nu_t = 0$.

Let us consider the representation obtained by acting with Λ on the reduced form VAR representation in Eq. (8)

$$\Lambda' A(L) Y_t = \Lambda' \nu_t \ . \tag{23}$$

Eq. (23) is a 'partially' identified SVAR of the form

$$\Lambda' Y_t = \sum_{j=1}^{k} \Lambda' A_j Y_{t-j} + \begin{pmatrix} \kappa u_t^i \\ \xi_t \end{pmatrix} \ . \tag{24}$$

A partially-identified MA is obtained by pre-multiplying Eq. (23) for $A(L)^{-1}\Lambda^{-1}$, where $\Lambda^{-1} = \Lambda = \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix}$, to get

$$Y_t = C(L) \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix} \begin{pmatrix} \kappa u_t^i \\ \xi_t \end{pmatrix} = \kappa C(L) \lambda u_t^i + C(L)\tilde{\lambda}\xi_t \ . \tag{25}$$

□

A few observations are in order. First, Eq. (25) implies that the Wold moving average can be factorised into two terms. The first one depends on the invertible shock u_t^i, and the second one is a function of the $n-1$ linear combinations of the Wold innovations orthogonal to u_t^i. It is worth noticing that while the requirement that ξ_t and u_t^i are orthogonal is important, we do not require ξ_t to span the space of all the shocks orthogonal to u_t^i.

Second, the above result implies that if the VAR has a correctly specified lag order, under partial invertibility the 'partially' identified SVAR impulse response functions are the dynamic causal effects to the identified shock u_t^i.

Third, the argument above can be readily extended to λ of dimension $n \times m$ for $m < n$. In fact, Proposition 1 readily generalises to the case of m partially invertible structural shocks, for $m > 1$. In such a case, the first term of the semi-structural Wold moving average depends on the m partially invertible shocks, and the second is a linear combination of the lags and leads of the remaining $n - m$ shocks.

4 IV Identification under Partial Invertibility

Let us consider a partially invertible VAR with reduced form representation as in Eq. (8), repeated below for convenience

$$A(L)Y_t = \nu_t \qquad A_0 = \mathbb{I}_n \ . \tag{8}$$

Given an external instrument z_t, it is possible to identify u_t^1 and its effects on $Y_{t+h},\ h = 0, \ldots, H$, under the set of conditions in the following proposition.

Proposition 2. Identification in SVAR-IV under Partial Invertibility *Let z_t be an instrument for the shock u_t^1 that satisfies the following conditions:*

(i) $\mathbb{E}[u_t^1 z_t'] = \alpha$ *(Relevance)*

(ii) $\mathbb{E}[u_t^{2:n} z_t'] = 0$ *(Contemporaneous Exogeneity)*

(iii) $\mathbb{E}[u_{t+j}^k z_t'] = 0$ *for all $j \neq 0$ and $k \neq 1$ such that $E[u_{t+j}^k \nu_t'] \neq 0$. (Limited Lead-Lag Exogeneity)*

The impact effect λ of u_t^1 onto Y_t is identified (up to a scale) as

$$\lambda \propto \mathbb{E}[\nu_t z_t'].$$

Proof. Let u_t^1 be a partially invertible structural shock such that $Y_t = \kappa C(L)\lambda u_t^1 + C(L)\tilde{\lambda}\xi_t$, where ξ_t is a linear combination of leads and lags of the remaining $(n-1)$ structural shocks $u_t^{2:n}$, some of which may be non-invertible, and λ, $\tilde{\lambda}$ are defined as in Proposition 1. Conditions (i) to (iii) imply that

$$\mathbb{E}[\nu_t z_t'] = \mathbb{E}\left[\begin{pmatrix}\lambda & \tilde{\lambda}\end{pmatrix}\begin{pmatrix}\kappa u_t^1 \\ \xi_t\end{pmatrix} z_t'\right] = \begin{pmatrix}\lambda & \tilde{\lambda}\end{pmatrix}\begin{pmatrix}\kappa\mathbb{E}[u_t^1 z_t'] \\ \mathbb{E}[\xi_t z_t']\end{pmatrix} = \alpha\kappa\lambda \ .$$

□

The above conditions (i) and (ii) are the conventional relevance and exogeneity conditions for instrumental variables (IV) that are standard in the micro and macro literatures (see Stock and Watson, 2018). Condition (iii) arises because of the dynamics, and it requires that if there are any non-invertible shocks, they do not correlate with the instrument at any leads and lags. Conversely, leads and lags (but not contemporaneous values) of other partial invertible shocks can contaminate the instrument without compromising the identification of λ.

If the system is invertible and the VAR correctly captures the data generating process of Y_t, then the third condition is trivially satisfied, since ν_t are a linear combination only of the contemporaneous structural shocks u_t. Conversely, when all the remaining shocks are non-invertible, Condition (iii) implies a stronger lead-lag exogeneity condition (i.e. $\mathbb{E}[u_{t+j}^k z_t'] = 0$ for all $j \neq 0$ and for all $k \neq 1$) that applies to all the shocks but the partially invertible one.

In the more general case in which only some of the remaining shocks are non-invertible, Proposition 2 ensures that identification with an external instrument is possible as long as the instrument is contaminated only by the past and future realisations of the invertible shocks. These are the shocks that do not enter the VAR innovations ν_t.

Condition (iii) is a relatively stronger condition than that required for a well specified and fully invertible SVAR (where lead-lag exogeneity is not required), but still a much weaker one than the lead-lag exogeneity condition required for identification in direct regressions and LP-IV.

When Condition (iii) is violated, the instrument is contaminated by leads and/or lags of some of the non-invertible shocks. This results in a bias in the estimated impulse response functions; we formalise this observation in the following remark.

Remark 1. (Violation of the Exogeneity Condition) *Let z_t be an instrument for a partially invertible shock u_t^1 that satisfies Condition (i) but fails Condition (ii) and Condition (iii) of Proposition 2, due to contamination by lags, leads or contemporaneous realisations of a non-invertible shock $u_t^{\not{1}}$, i.e.*

$$z_t = \alpha u_t^1 + \sum_k \beta_k u_{t-k}^{\not{1}} \ , \tag{26}$$

for $k \in \mathbb{Z}$. Given a well specified VAR, the innovations of the Wold representation can be mapped into the structural shocks as

$$\nu_t = (b^{(1)} \ \ b^{(2)}(L)) u_t \ , \tag{27}$$

where $b^{(1)}$ is a $n \times 1$ matrix and $b^{(2)}(L)$ is a $n \times (n-1)$ matrix lag polynomial that incorporates Blaschke factors, due to the presence of non-invertible shocks. The estimated IRFs for variable i, to shock 1, at horizon h, are biased and of the form

$$\widetilde{IRF}_{i1}^h = IRF_{i1}^h + \left[C_h \sum_j \sum_k b_{j,\not{1}}^{(2)} \frac{\beta_k}{\alpha} \delta_{jk} \right]_i \ , \tag{28}$$

where IRF_{i1}^h are the IRFs at horizon h to the shock u_t^1 and the second term is a bias. C_h are the matrix coefficients of the Wold representation at lag h, and $b_{j,\not{1}}^{(2)}$ is the $\not{1}$ column of the matrix of coefficients of the polynomial at lag j. δ_{jk} is the Kronecker's delta.

Proof. Given a well specified VAR, the Wold representation is

$$Y_t = C(L)\nu_t \ , \tag{29}$$

and the semi-structural moving average representation is of the form

$$Y_t = \kappa C(L)\lambda u_t^i + C(L)\tilde{\lambda}\xi_t \ . \tag{30}$$

We can write the Wold residuals as

$$\nu_t = (\kappa\lambda \ \ \tilde{\lambda}) \begin{pmatrix} u_t^1 \\ \xi_t \end{pmatrix} = \begin{pmatrix} b^{(1)} & b^{(2)}(L) \end{pmatrix} u_t \tag{31}$$

where $b^{(1)}$ is a $n \times 1$ matrix and $b^{(2)}(L)$ is a $n \times (n-1)$ matrix lag polynomial that incorporate Blaschke factors, due to the presence of non-invertible shocks. In this case

$$\mathbb{E}[\nu_t z_t'] = \mathbb{E}\left[\begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix} \begin{pmatrix} \kappa u_t^1 \\ \xi_t \end{pmatrix} z_t' \right] = \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix} \begin{pmatrix} \kappa \mathbb{E}[u_t^1 z_t'] \\ \mathbb{E}[\xi_t z_t'] \end{pmatrix} = b^{(1)} + \sum_j \sum_k b_{j,\not 1}^{(2)} \beta_k \delta_{jk} \ ,$$

where the Kronecker's delta singles out the common lags in $u_t^{\not 1}$ between the instrument and the column $\not 1$ of the matrix lag polynomial $b^{(2)}(L)$. By normalising for the coefficient of correlation α and multiplying for the matrix C_h of lag h of the Wold representation one finds

$$\widetilde{IRF}_{i1}^{h} = \left[C_h b^{(1)} + C_h \sum_j \sum_k b_{j,\not 1}^{(2)} \frac{\beta_k}{\alpha} \delta_{jk} \right]_i \ ,$$

which is the expression in Eq. (28). □

A few elements of Eq. (28) are worth highlighting. First, all else equal, the amount of bias in the estimated IRFs depends on how much the instrument correlates with the (leads and lags of the) contaminating shock as compared to the shock of interest – i.e. on the ratios $\frac{\beta_k}{\alpha}$. Second, the bias depends on the number of lags that are common to those contaminating the instrument (Eq. 26) and those that appear in the Blaschke polynomial $b^{(2)}(L)$. Finally, and importantly, the bias depends on the relative order of magnitude of the coefficients $b^{(2)}_{j,\not{1}}$ as compared to $b^{(1)}$. These relate to the variance of variable i that is accounted for by the shock of interest and the contaminating shocks. For example, very small values of $b^{(2)}_{j,\not{1}}$ relative to $b^{(1)}$ imply that shock $\not{1}$ explains very little of the variation in variable i, and hence the distortion is likely to be small. While the contamination of the instrument biases both the impact and the dynamic responses, in the next section we discuss how under partial invertibility, the impact effects of the shock of interest can be correctly recovered also in misspecified VARs, as long as Condition (iii) holds.

5 An Observation on VAR Misspecifications

Let us consider a purely nondeterministic, stationary VARMA(p,q) process $Y_t = (y'_{1,t} \;\; y'_{2,t})'$

$$\begin{pmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{pmatrix} \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \Psi_{11}(L) & \Psi_{12}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) \end{pmatrix} \begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix}. \tag{32}$$

Fitting a VAR(k) to $y_{1,t}$ corresponds to imposing some or all of the following restrictions

$$\Phi_{11,i} = 0, \qquad i = k+1, k+2, \ldots p, \tag{33}$$

$$\Phi_{12,i} = 0, \qquad i = 1, 2, \ldots p, \tag{34}$$

$$\Psi_{11,i} = 0, \qquad i = 1, 2, \ldots q, \tag{35}$$

$$\Psi_{12,i} = 0, \qquad i = 1, 2, \ldots q. \tag{36}$$

Let us consider the case in which only some of these restrictions are not reflected in the data generating process. The first restriction (conditional on the others being true) corresponds to understating the VAR lag order with $k < p$. The second restriction would instead imply the exclusion of relevant variables. This is also a trivial case of non-invertibility due to the number of variables being smaller than the number of shocks. Finally, the last two restrictions correspond to disregarding the MA structure of the process. Braun and Mittnik (1993) discuss and quantify the asymptotic biases resulting from these misspecifications.

We now consider what these misspecifications imply for the identification of a shock of interest u_t^i, under the assumption of partial invertibility. Let us assume that a condition of partial invertibility for u_t^i on the subvector $y_{1,t}$ holds, i.e.

$$u_t^i = Proj(u_t^i | y_{1,t}, y_{1,t-1}, \dots) \ . \tag{37}$$

This condition guarantees that u_t^i can be obtained as linear projection of $y_{1,t}$ onto its lags (potentially infinitely many).

Let us now consider the case of a too short lag order. In this case, the autoregressive coefficients would be biased and inconsistent. However, if the system contains sufficiently many lags to fulfil the partial invertibility condition in Eq. (37), then identification is still obtained. Hence, while impact responses of the variables to the shocks of interest are correctly estimated, their dynamics are distorted even asymptotically. Exactly the same logic applies to the case of a misspecified moving average component, that can always be mapped into a VAR with infinitely many lags. It is worth observing that while in the first case more lags trivially resolve the issue, in the second case longer lags only asymptotically approximate the correct Wold representation.

Consider now the case of omitted variables. Also in this case, if partial invertibility in Eq. (37) for the subset of variables $y_{1,t}$ holds, then impact coefficients are correctly retrieved while the IRFs are be distorted. However, interestingly, also in this case longer lags would asymptotically capture the correct dynamics of the system. To see this, consider the following. The Wold Representation Theorem implies that also $y_{1,t}$ has an invertible MA representation. For the n_1-dimensional subprocess $y_{1,t} = JY_t$, where $J_t = (\mathbb{I}_{n_1} \;\; 0_{n-n_1})$ is a selector matrix, we can write

$$\Phi_{11}(L)y_{1,t} = -\Phi_{12}(L)y_{2,t} + \Psi_1(L)u_t \; . \tag{38}$$

If Y_t is covariance-stationary, $y_{1,t}$ is also covariance stationary, with first and second moments respectively equal to $\mathbb{E}(y_{1,t}) = J\mathbb{E}(Y_t)$, and $\Gamma_{y_1}(h) = J\Gamma_Y(h)J'$, where $\Gamma(h)$ is the autocovariance of Y_t at lag h. The Wold Representation Theorem also guarantees the existence of an ARMA representation of the form

$$\widetilde{\Phi}_1(L)y_{1,t} = \widetilde{\Psi}_1(L)\nu_{1,t} \; . \tag{39}$$

The true innovations u_t are trivially non-invertible in $y_{1,t}$. In fact, the n innovations u_t are compounded and reduced to the $n_1 < n$ innovations $\nu_{1,t}$ which do not have a meaningful structural interpretation. If system is partially invertible in $y_{1,t}$ then impact of the shock of interest are correctly estimated, moreover the existence of a Wold representation guarantees that the dynamics of the system is asymptotically approximated by infinitely many lags of $y_{1,t}$ only.

While in all of the cases discussed VARs can only asymptotical approximate the true dynamics of the system, direct methods à la Jordà (2005) with controls can be used to improve over VAR estimates.

Interestingly, these observations provide a simple way to gauge the contamination of an instrument versus the misspecification of the model adopted – two dimensions along which structural identification may be problematic and deliver unstable results. In fact, if one can assume partial invertibility across different specifications of an empirical model, an instrument fulfilling the conditions for identification would deliver stable impact responses but unstable IRFs across models. In this case, increasing the number of lags and/or selectively adding variables that may be of importance for the transmission of the shock should help stabilise the dynamics response. The intuition for this is that additional controls may be important for the transmission of the structural shocks (see discussion in Caldara and Herbst, 2018). Conversely, an instrument that violates the lead-lag exogeneity conditions is likely to deliver also unstable impact response across different models.[10] We provide empirical support to these remarks in the following sections.

6 Partial Invertibility in a Simulated System

We simulate data from a stylised New Keynesian DSGE model that features (*i*) a representative infinitely-lived household that chooses between consumption and leisure; (*ii*) firms that produce a continuum of goods using a Cobb-Douglas technology to aggregate capital and labour; (*iii*) a government that consumes a share of output for wasteful public spending; and (*iv*) a central bank that sets the interest rate using a Taylor rule with smoothing. There are four stochastic disturbances that generate fluctuations in the economy, namely, a monetary policy shock u_t^r, a government spending shock u_t^g, a technology shock u_t^a, and an inflation-specific shock u_t^π.

The processes for technology, spending, prices, and the policy rate are defined as follows. Log technology a_t evolves with a news component as

$$a_t = \rho_a \, a_{t-1} + \omega u_{t-4}^a \ , \tag{40}$$

where u_t^a is an i.i.d. normally distributed technology shock. Similarly, an element of fiscal foresight characterises the spending process g_t, that evolves according to

$$g_t = \rho_g \, g_{t-1} + u_{t-4}^g \ , \tag{41}$$

where u_t^g is an i.i.d. normally distributed spending shock. The monetary authority

[10]In this case, a much larger information set can help resolving the issue. The intuition for this is that structural shocks are likely to be fundamental and partially invertible in larger models, and can hence improve the performance of contaminated instruments. (see Giannone and Reichlin, 2006).

sets the nominal interest rate using a Taylor rule with smoothing

$$r_t = \rho_r \, r_{t-1} + (1-\rho_r)\left(\phi_\pi \overline{\pi_t} + \phi_y \overline{\Delta y_t}\right) + u_t^r \ , \tag{42}$$

where $\overline{\pi_t}$ is the average inflation over the last four periods, $\overline{\Delta y_t}$ is the average growth rate of output, and u_t^r is a white noise i.i.d. normally distributed monetary policy shock. Finally, price dynamics are governed by a New Keynesian Phillips Curve, as follows

$$\pi_t = \gamma_\pi \pi_{t-1} + \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\theta_\pi)(1-\theta_\pi\beta)}{\theta_\pi} mc_t + u_t^\pi, \tag{43}$$

where mc_t are marginal costs, and u_t^π is an i.i.d. normally distributed inflation-specific shock. All the model details, including the calibrated parameters, are reported in Appendix A.

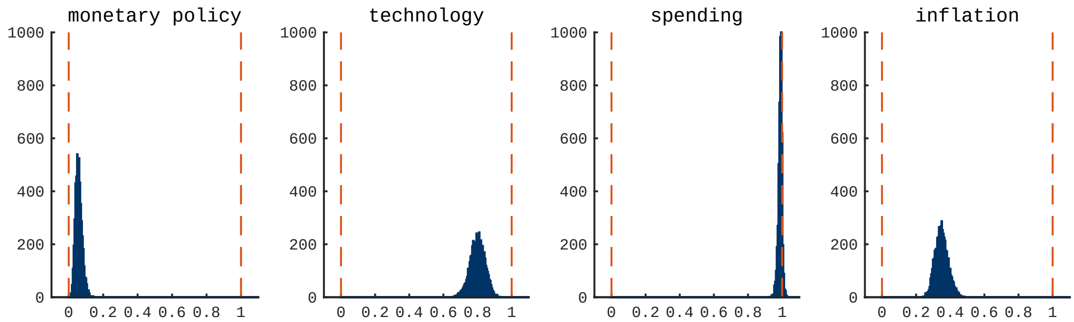
We consider a VAR(4) in the policy rate, inflation, output, and spending. Under the chosen set of parameters, the model fails the 'poor man's invertibility condition' of Fernandez-Villaverde et al. (2007), hence, the four structural shocks cannot all be recovered from a VAR in the observables. However, the specification of the Taylor rule ensures the monetary policy shock is partially invertible from a VAR(4) in $[r_t, \pi_t, y_t]'$.

Figure 1 reports the degree of invertibility δ_u of each of the structural shocks in the model. From the model, we simulate 5,000 economies each of sample size $T = 300$ periods. For each set of simulated data, we then estimate a VAR(4) in the four observables (output growth, inflation, government spending and the policy interest rate) and calculate the degree of invertibility of each of the four shocks as in Forni et al. (2018)

$$\delta_u = \text{var}[u_t^i - Proj(u_t^i | \mathcal{H}_t^Y)]/\sigma_{u_i}^2, \tag{44}$$

where $\sigma_{u_i}^2$ denotes the variance of the shock of interest. Hence, δ_u is a measure of the unexplained variance of the orthogonal projection of each of the structural shocks onto the VAR residuals. A value of 0 implies that the shock is recoverable from the VAR, whereas increasing values of δ_u imply non-fundamentalness and an increasing

Figure 1: Degree of Invertibility of the Structural Shocks



Note: Distribution of δ_u across 5000 simulated economies. $\delta_u = 0$ denotes invertibility; $\delta_u = 1$ denotes insufficient information for shocks recoverability. VAR(4).

Table 1: Variance Decomposition

		u_t^r	u_t^a	u_t^g	u_t^π
output	y_t	16.5	75.56	0.38	12.71
spending	g_t	0.00	0.00	68.09	0.00
inflation	π_t	9.05	51.23	0.03	66.89
policy rate	r_t	25.96	20.31	0.04	14.84

Note: Share of variance accounted for by each shock. Numbers may not add up to 100 due to non-zero correlation of simulated shocks in small samples.

degree of non-recoverability. Across simulations, the distribution of δ_u for technology and spending is strongly concentrated towards the upper bound of 1, confirming the inability of the VAR to recover these two shocks. Similarly, the inflation shock is also non-invertible, but with a higher degree of recoverability. The four shocks play a different role in driving economic fluctuations in the model. Table 1 reports the share of variance of the four observables that is accounted for by each of the four shocks in the model. We note that the government spending shock plays a negligible role.

We now use the same set of VAR(4) on the simulated data to identify the monetary policy shock using the following four different external instruments:

$$z_{0,t} = u_t^r \ , \tag{45}$$

$$z_{1,t} = 0.7u_t^r - 0.5u_{t-2}^r + \varsigma_t \ , \tag{46}$$

$$z_{2,t} = 0.7u_t^r - 0.5\left(u_{t-1}^g + u_{t-2}^g + u_{t-3}^g\right) + \varsigma_t \ , \tag{47}$$

$$z_{3,t} = 0.7u_t^r + 0.5\left(u_{t-1}^a + u_{t-2}^a + u_{t-3}^a\right) + \varsigma_t \ . \tag{48}$$

In Eq. (45) the shock is perfectly observable. This is the case discussed in Stock and Watson (2018). The instrument in Eq. (46) is an instrument contaminated by classic white noise measurement error, and the second lag of the monetary policy shock. The instruments in Eqs. (47-48) both fail the lead-lag exogeneity condition. In fact, while $z_{2,t}$ is contaminated by lagged spending shocks, $z_{3,t}$ correlates with lagged technology shocks. In all cases, ς_t is a normally distributed random measurement error with zero mean and variance equal to that of the structural shocks. A VAR(4) allows for partial invertibility and also captures the model's dynamics sufficiently well. Hence, we use $p = 4$ as the benchmark case.[11]

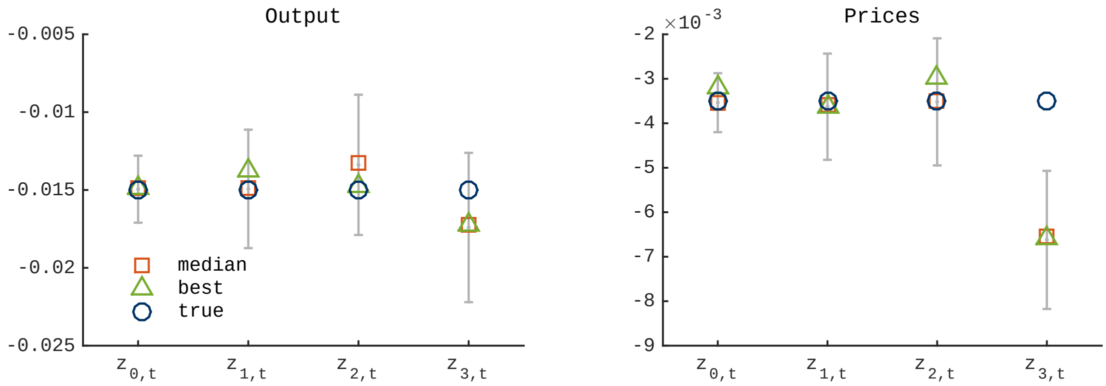
Impact responses for output and inflation recovered from the four instruments and a VAR(4) are in Figure 2.[12] In each subplot, we use blue circles for the model's responses (true), orange squares for the median across simulations, and green triangles for the simulation which is the closest to the median (best).[13] The error bars are two standard deviations intervals constructed from the distribution across simulations. A few elements are worth highlighting. As also noted in Stock and Watson (2018), when the shock is observable ($z_{0,t}$), the assumption of full invertibility can be dispensed with for the validity of SVAR-IV. However, the shock is correctly recovered also under the milder conditions introduced in Section 4. In fact, correct impact responses are recovered also with $z_{1,t}$. The introduction of a measurement error in $z_{1,t}$ widens the

[11] In the Appendix we also report the extreme cases of $p = 1$ and $p = 2$ where the model is more severely misspecified and the identification becomes more challenging.

[12] IRFs are normalised such that the impact response of the policy rate to a monetary policy shock equals that of the model.

[13] We select the simulation whose IRFs minimise the sum of square deviations from median IRFs over the first 12 periods. The choice allows to put more weight at shorter horizons where responses display richer dynamics. Changing the truncation horizon yields qualitatively similar results.

Figure 2: Impact Responses to Monetary Policy Shock

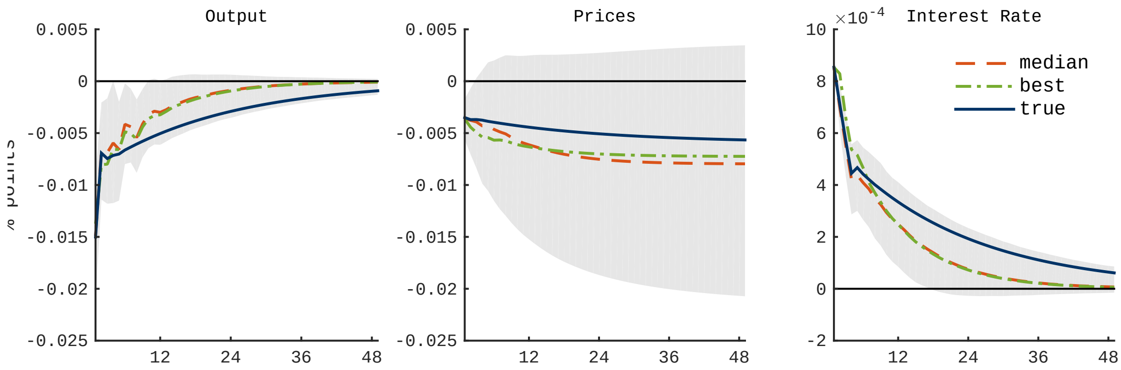


Note: Impact responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(4) in four observables. $z_{0,t}$: observed shock case; $z_{1,t}$: instrument correlates with monetary policy shock only; $z_{2,t}$: instrument also correlates with past spending shocks; $z_{3,t}$ instrument correlates also with past technology shocks. Grey vertical lines are 2 standard deviations error bards from the distribution of impact responses across 5,000 simulated economies of sample size $T = 300$ periods. True impact (blue circle), median across simulations (orange square), minimum distance from median (best) simulation (green triangle).

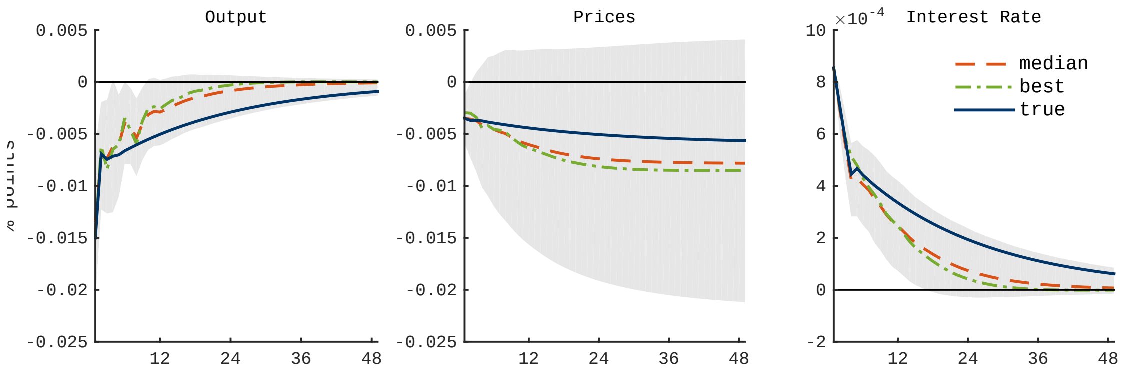
distribution of impact responses across simulations, but recovers the correct impact effects. The picture changes substantially when we consider the case of $z_{3,t}$. In this case, the instrument correlates with lagged non-invertible technology shocks which the data in the VAR cannot provide sufficient information for by construction. This results in severely biased impact responses. An interesting case arises when the instrument also correlates with lagged spending shocks ($z_{2,t}$). The spending shock is not invertible in the system, however, as noted, it is responsible for a negligible share of the variance of the simulated variables. In this case the contamination is ineffective, and impact responses are correctly recovered.

The discussion extends in an equivalent way to responses at farther away horizons. Figure 3 reports responses estimated over 48 periods using $z_{1,t}$ (Panel A, top), $z_{2,t}$ (Panel B, centre), and $z_{3,t}$ (Panel C, bottom). In the first two cases the model responses lie within the bands generated across the simulations. On the contrary, the response of all variables are outside the simulation confidence region when the shock is identified using $z_{3,t}$.

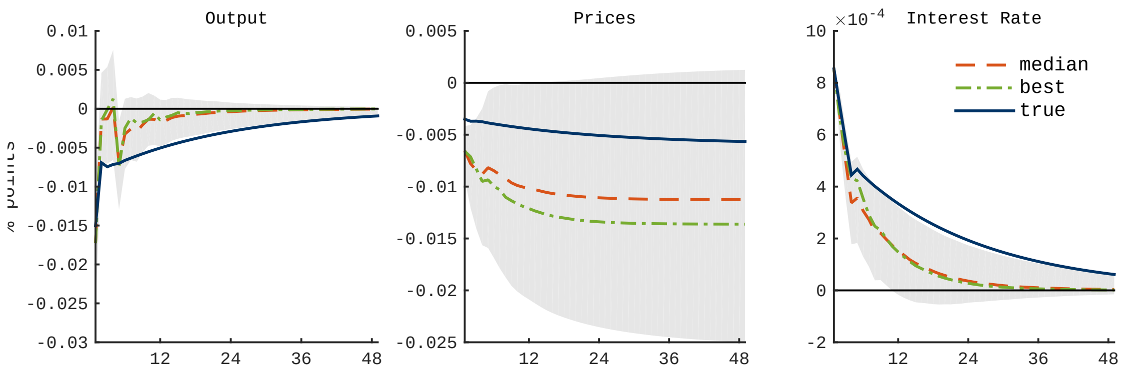
Figure 3: Responses to Monetary Policy Shock – Simulation



(A) $z_{1,t}$: external instrument correlates with monetary policy shock only



(B) $z_{2,t}$: external instrument also correlates with lagged spending shocks



(C) $z_{3,t}$: external instrument also correlates with lagged technology shocks

Notes: Impulse responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(4) in four observables. Instrument correlates with monetary policy shock only (Panel A). Instrument correlates with monetary policy shocks and lagged spending shocks (Panel B). Instrument correlates with monetary policy shocks and lagged technology shocks (Panel C). Grey shaded areas denote 90th quantiles of the distribution of IRFs across 5,000 simulated economies of sample size $T = 300$ periods. Model responses (true, blue solid), median across simulations (orange dashed), minimum distance from median (best) simulation (green dash-dotted).

This exercise shows that in a simulated environment full invertibility is not necessary for the identification of SVAR-IV. Moreover, under partial invertibility of the shock of interest, the impact responses are correctly estimated provided that the conditions laid out in Section 3 hold. Finally, when the limited lead-lag exogeneity condition is violated, the extent of the distortion depends on the share of variance of the variables that is accounted for by the shock that contaminates the instrument.

7 IV Identification of Monetary Policy Shocks

In this section, we look at the empirical identification of monetary policy shocks and use the results in the previous sections to shed light on the distortions to the impact and dynamic responses that arise from either the contamination of the instrument, or the misspecification of the VAR model. In particular, we consider different potential instrument for monetary policy shocks, some of which may be contaminated, and assess dependence of the impact and dynamics responses on different information sets and/or VAR specifications, some of which are likely to be misspecified.

We consider three external instruments, all of which are constructed from the high-frequency surprises of Gürkaynak et al. (2005) and measure monetary policy innovations through the surprise reactions of federal funds futures markets around FOMC announcements, following the insight of Kuttner (2001). The first of these instruments is constructed by measuring high-frequency surprises around all the scheduled FOMC meetings between 1990 and 2012. This is equivalent to the instrument used in e.g. Stock and Watson (2018) and Caldara and Herbst (2018), and we denote it by $z_{\mathrm{A},t}$. The second instrument is a monthly moving average of high-frequency surprises around all FOMC announcements from 1990 to 2012. This is the instrument originally proposed in Gertler and Karadi (2015), denoted $z_{\mathrm{B},t}$. The third external instrument – $z_{\mathrm{C},t}$ – is the residual of a projection of high-frequency surprises around all FOMC meetings onto their lags and Fed Greenbook forecasts from 1990 to 2009 (see Miranda-Agrippino and Ricco, 2017). This projection can be seen as a pre-whitening

Table 2: Contamination of Monetary Policy Instruments

$H_0 : \beta_{f_{1,t-1}} = \beta_{f_{2,t-1}} = \ldots = \beta_{f_{10,t-1}} = 0$			
	$z_{\mathrm{A},t}$	$z_{\mathrm{B},t}$	$z_{\mathrm{C},t}$
$F_{(10,227)}$	2.12 (0.0240)		
$F_{(10,226)}$		3.52 (0.0002)	
$F_{(10,215)}$			1.77 (0.0669)
N	239	238	227

Note: Wald test statistics. Regressions include a constant and one lag of the dependent variable. Sample 1990:2009. p-values in parentheses.

step that removes contamination with other past and contemporaneous shocks related to the state of the economy, due to the signalling channel of monetary policy.[14] All three instruments are monthly, and use the fourth federal funds futures as underlying contracts to measure the high-frequency surprises.

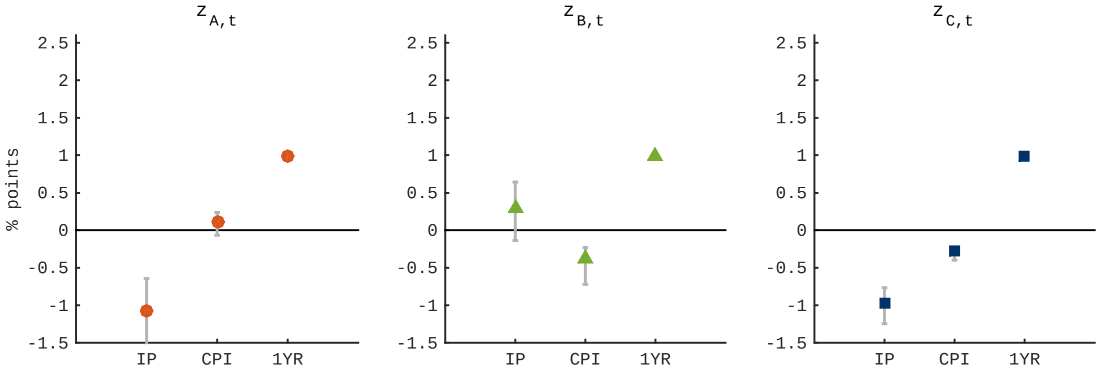
Table 2 reports Granger causality tests for the three instruments on the first ten macroeconomic and financial factors estimated from the monthly dataset in McCracken and Ng (2015), that include a constant and one lag of the selected instrument. The numbers in the table are Wald test statistics for the null that the factors' coefficients are jointly equal to zero.[15] Results point to possible contamination of the $z_{\mathrm{A},t}$ and $z_{\mathrm{B},t}$ instruments by lagged macroeconomic shocks, with p-values well beyond the rejection region. This serves as motivation for our next exercise.

We evaluate the effect of the instruments' contamination on the estimation of the IRFs in an empirical setup that encompasses standard monetary VARs such as those in Coibion (2012) and Gertler and Karadi (2015). Our benchmark VAR is monthly and estimated with 12 lags from 1979:1 to 2012:12. The variables included are the one-

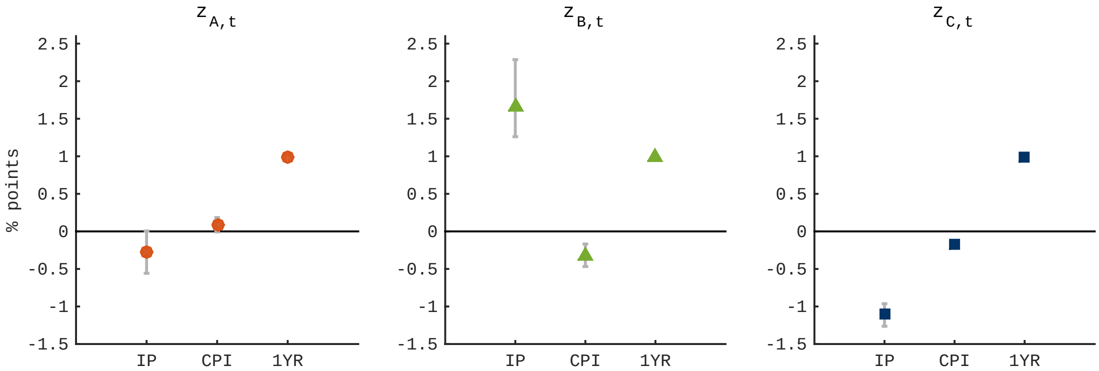
[14]The intuition for this is that the policy rate announcements can signal the central bank's view about macroeconomic developments to market participants. This implies that market price revisions incorporate both the monetary policy shocks and the information update about the state of the economy (see Melosi, 2017 and Miranda-Agrippino and Ricco, 2017).

[15]Full regression results are reported in the Appendix.

FIGURE 4: IMPACT RESPONSES TO MONETARY POLICY SHOCKS – 1979:2012



(A) Baseline VAR



(B) Misspecified VAR

Notes: Baseline VAR(12) in all variables, top panel (A). Misspecified VAR(2) in three variables, bottom panel (B). VARs estimated with standard macroeconomic priors. Identification in all cases uses the full length of the instruments. $z_{\text{A},t}$: high-frequency surprises at scheduled FOMC meetings; $z_{\text{B},t}$: moving average of high-frequency surprises within the month; $z_{\text{C},t}$: residuals of $z_{\text{A},t}$ on Fed Greenbook forecasts. Shaded areas denote 90% posterior coverage bands.

year government bond rate as the policy variable, an index of industrial production, the unemployment rate, the consumer price index, a commodity price index, and the excess bond premium (EBP) of Gilchrist and Zakrajšek (2012).[16] Stock and Watson (2018) show that in this system there is no statistically significant evidence against the null hypothesis of invertibility.[17]

We also consider a VAR which omits the unemployment rate, the EBP variable, and the commodity price index, includes only 2 lags, and is hence likely to be misspecified. In all cases, we estimate the impact responses from a regression of the VAR innovations onto one of the above instruments, while IRFs are retrieved from the coefficients of the VAR. Responses are normalised such that the policy rate increases by 1% on impact.

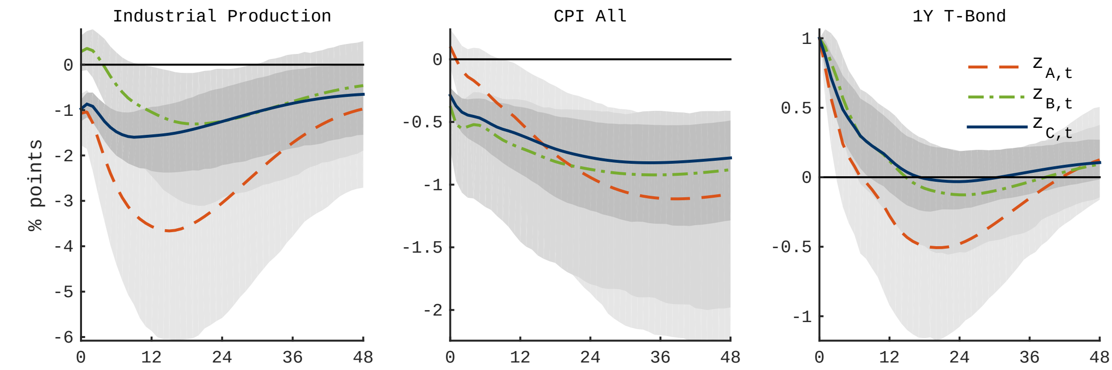
We start by looking at the impact responses retrieved by the three instruments in the two VARs, reported in Figure 4. The top row collects results for the baseline VAR, while the misspecified VAR is in the bottom row. Comparing the impact responses for each given instrument across VARs we note that while $z_{\mathrm{C},t}$'s impact are stable, impact responses under either $z_{\mathrm{A},t}$ or $z_{\mathrm{B},t}$ vary and are statistically different. Modal impact responses of production to a contractionary monetary policy shock go from being non significant to strongly positive at almost 2% under $z_{\mathrm{B},t}$, and from -1% to essentially zero under $z_{\mathrm{A},t}$. The impact response under $z_{\mathrm{C},t}$ is largely unchanged. The impact response of prices under $z_{\mathrm{A},t}$ is slightly positive.

We then turn to the full dynamic responses reported in Figure 5. Notwithstanding the differences in the impact responses just discussed, the responses in the baseline VAR are qualitatively coherent; all instruments identify a monetary policy shock that eventually triggers an economic recession, accompanied by a significant contraction in prices. However, the picture changes quite materially as we move to the misspecified

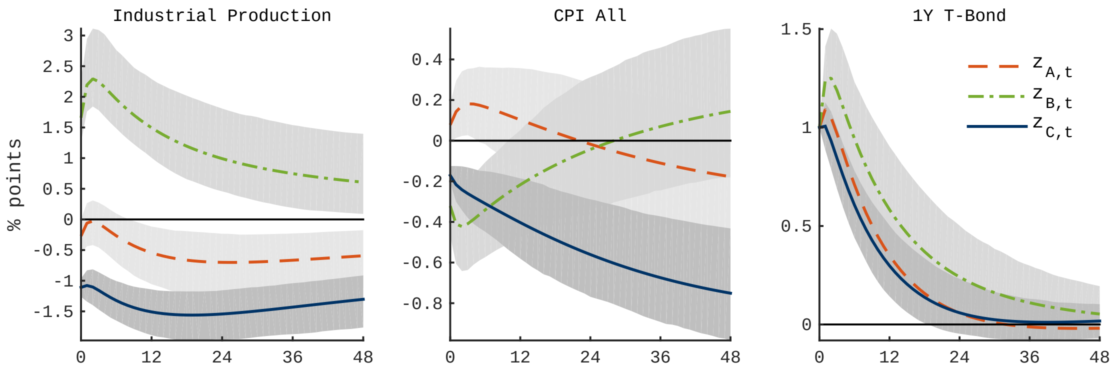
[16]Data for bond yields, industrial production, and the consumer price index are from the St Louis FRED Database, the commodity price index is from the Commodity Research Bureau, the EBP data are from the Federal Reserve Board.

[17]Stock and Watson (2018) do not reject the null of invertibility in a system that includes industrial production, the index of consumer prices, the one year interest rate and the excess bond premium variable.

FIGURE 5: RESPONSES TO MONETARY POLICY SHOCKS – 1979:2012



(A) Baseline VAR



(B) Misspecified VAR

Notes: Baseline VAR(12) in all variables, top panel (A). Misspecified VAR(2) in three variables, bottom panel (B). VARs estimated with standard macroeconomic priors. Identification in all cases uses the full length of the instruments. $z_{\mathrm{A},t}$: sum of high-frequency surprises within the month; $z_{\mathrm{B},t}$: moving average of high-frequency surprises within the month; $z_{\mathrm{C},t}$: residuals of $z_{\mathrm{A},t}$ on Fed Greenbook forecasts. Shaded areas denote 90% posterior coverage bands.

VAR (bottom row of Figure 5).[18]

These results suggests that neither $z_{\mathrm{A},t}$ nor $z_{\mathrm{B},t}$ satisfy the limited lead-lag exogeneity condition, i.e. they correlate with other shocks, likely related to developments in financial markets and the real economy, that the trivariate VAR(2) is not able to

[18] These findings are confirmed across different samples, as we show in the VARs in the Appendix (Figure B.4) estimated from 1990, date that coincides with the start date of all the instruments used.

control for.[19,20] Interestingly, dynamic responses obtained with $z_{\mathrm{C},t}$ are largely similar across the two specifications, pointing to a small degree of model misspecification.

8 Conclusions

This paper provides conditions for identification with external instruments in Structural VARs under partial invertibility. Partial invertibility is a general and not very stringent condition that is required when only one or a subset of the structural shocks in the system are of interest. Results show that SVAR-IV methods (and LP-IV with controls) allow for identification of the dynamic causal effects of interest under the standard relevance and contemporaneous exogeneity conditions plus a limited lead-lag exogeneity condition. The latter implies that the instrument can be contaminated by other partially invertible shocks (at different lags and leads) and still achieve correct identification. These conditions are weaker that the standard full invertibility condition often required for SVAR-IV, or the strong lead-lag exogeneity condition needed for LP-IV without controls. Hence, they extend the range of empirical settings in which SVAR-IV and LP-IV with controls can be used. Lastly, we show that identification of impact effects is possible even in the presence of model misspecification. In this case, an empirical trade-off between efficiency and accuracy of the impulse response functions arises and the use of larger information sets or of direct methods can help producing more robust inference.

[19]The first factor used in Table 2 is typically regarded as a synthetic measure of real activity, see e.g. McCracken and Ng (2015). Other than a barometer for financial markets' health levels, the EBP has strong predictive powers for an array of measures of economic activity, and its inclusion is likely to account for other omitted variables too (see e.g. Gilchrist and Zakrajšek, 2012; Gertler and Karadi, 2015).

[20]These results are invariant to a number of robustness tests, including on the estimation sample and the use of scheduled FOMC meetings only, as discussed extensively in Miranda-Agrippino and Ricco (2017).

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A Model

The economy is populated by a representative infinitely-lived household seeking to maximise

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t) \ , \tag{A.1}$$

with a period utility

$$U(C_t, H_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{H_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \ , \tag{A.2}$$

where σ is the risk aversion parameter, φ is the Frisch elasticity, and H_t are hours worked. C_t is a consumption bundle defined as

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} \right)^{\frac{\varepsilon}{1-\varepsilon}} \ , \tag{A.3}$$

where $C_t(i)$ is the quantity of good i consumed by the household in period t. A continuum of goods $i \in [0, 1]$ exists. The log-linearised households optimality conditions are given by the Euler equation

$$c_t = \mathbb{E}[c_{t+1}] - \frac{1}{\sigma} \left(r_t - \mathbb{E}[\pi_{t+1}] \right) \ , \tag{A.4}$$

and by the labour supply schedule

$$w_t = \frac{1}{\varphi} h_t + \sigma \, c_t \ , \tag{A.5}$$

where w_t is the labour wage on a competitive labour market. Agents maximise their intertemporal utility subject to a flow budget constraint. Agents can hold bonds or firms capital, and a no arbitrage condition between bonds and capital holds

$$\frac{1}{\beta} \left(r_t - \mathbb{E}[\pi_{t+1}] \right) = \frac{1}{\beta - (1-\delta)} \mathbb{E}[z_{t+1}] \ , \tag{A.6}$$

where δ is the rate of depreciation of capital. Firms produce differentiated goods $j \in [0, 1]$ by using a Cobb-Douglas technology to aggregate capital and labour

$$Y_t(j) = A_t K_{t-1}(j)^{\alpha} H_t(j)^{1-\alpha} \tag{A.7}$$

where, importantly, log technology $a_t \equiv log(A_t)$ has a news component

$$a_t = \rho_a \, a_{t-1} + \omega u^a_{t-4} \ , \tag{A.8}$$

where u^a_t is an i.i.d. normally distributed technology shock. The static optimality condition on the production inputs delivers the linearised relation

$$w_t + h_t = k_{t-1} + z_t \ . \tag{A.9}$$

The log-linearised production function of the firms is

$$y_t = a_t + \alpha k_{t-1} + (1 - \alpha) \, h_t \ . \tag{A.10}$$

Firms set prices in a staggered way à la Calvo (1983) with an indexation mechanisms of the type proposed by Galì and Gertler (1999). Thus, each period, a measure $1-\theta$ of firms reset their prices, while prices for a fraction θ of the firms are $P_t(j) = P_{t-1}\pi^{\gamma}_{t-1}$. θ is an index of price stickiness. The firms that can reset their prices maximise the expect sum of profits

$$max_{P^*_t(j)} \sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} \left(P^*_t(j) \left(\frac{P_t - 1 + \tau}{P_{t-1}} \right)^{\gamma} - MC_{t+\tau} \right) Y_{t+\tau}(j) \ , \tag{A.11}$$

where MC_t are the real marginal costs in period t. The first order conditions from this problem, combined with the aggregate price equation, form a hybrid New Keynesian

Phillips Curve

$$\pi_t = \gamma\,\pi_{t-1} + \beta\mathbb{E}[\pi_{t+1}] + \lambda mc_t\ , \qquad \lambda \equiv \frac{(1-\theta)\ (1-\beta\,\theta)}{\theta} + u_t^{\pi}, \tag{A.12}$$

where u_t^{π} is an i.i.d. normally distributed inflation-specific shock, and marginal costs evolve as

$$mc_t = \alpha\, z_t + (1-\alpha)\, w_t - a_t\ . \tag{A.13}$$

The linearised law of motion for firms capital is

$$I_t = K_{t+1} - (1-\delta)K_t\ , \tag{A.14}$$

where K_t is physical capital and I_t is investment. The log-linearisation of this equation yields[21]

$$i_t = k_t - (1-\delta)\ k_{t-1}\ . \tag{A.15}$$

A fiscal authority absorbs a share of output into wasteful government spending

$$G_t = (1-\rho_g)G + \rho_g G_{t-1} e^{u_{t-4}^g} \tag{A.16}$$

and the log-linearised equation for government spending is

$$g_t = \rho_g\, g_{t-1} + u_{t-4}^g\ , \tag{A.17}$$

where u_t^g is an i.i.d. normally distributed government demand shock. At the steady state $G = gY$. A monetary authority sets the nominal interest rate using a monetary rule with a smoothing term

$$r_t = \rho_r\, r_{t-1} + (1-\rho_r)\left(\phi_\pi \overline{\pi_t} + \phi_y \overline{\Delta y_t}\right) + u_t^r\ , \tag{A.18}$$

[21]In order to have smoother impulse response functions, without introducing autocorrelation in the shock processes, we added an ad hoc quadratic adjustment of the form $i_t = k_t - (1-\delta)\ k_{t-1} + (k_t - (1-\delta)\ k_{t-1})^2$.

Table A.1: Calibrated Parameters

Parameter	Value	Description
α	0.4	share of capital in output
β	0.99	discount factor
δ	0.025	depreciation of capital
σ	1	risk aversion consumption
φ	2	labor disutility
g	0.2	share of public spending in output
θ	0.75	price stickiness
γ	0.2	indexation parameter (NK Phillips curve backward term)
ϵ	10	substitutability goods
ρ_r	0.95	monetary policy smoothing
ϕ_y	0.5	monetary policy output growth
ϕ_r	1.2	monetary policy inflation
ρ_a	0.5	productivity autocorrelation
ρ_g	0.95	public spending autocorrelation
ω	3	news multiplier

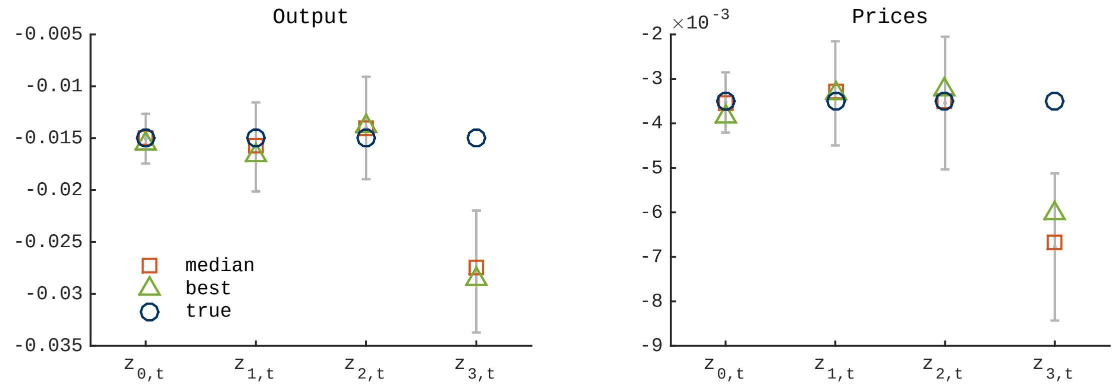
where $\overline{\pi_t}$ and $\overline{\Delta y_t}$ are, respectively, average inflation and the average rate of output growth over the last four periods, and u_t^r is a white noise i.i.d. normally distributed monetary policy shock. Importantly, the monetary policy innovation can be recovered from current and past values of the policy rate, inflation and output. Finally, the aggregate economy clears

$$Yy_t = Cc_t + Ii_t + Gg_t \ . \tag{A.19}$$

Table A.1 reports the calibration for this benchmark NK model. For this set of parameters the model fails the 'poor man's invertibility condition' of Fernandez-Villaverde et al. (2007).

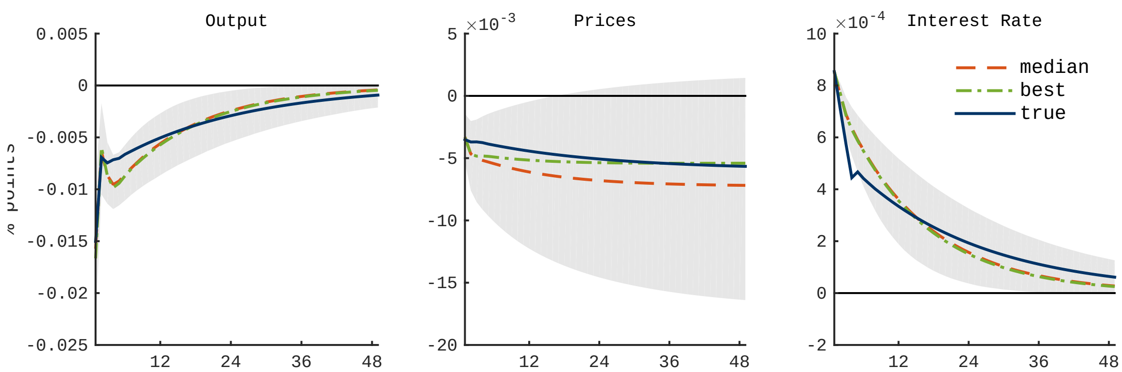
B Additional Charts & Tables

Figure B.1: Responses to MP Shock – Simulation & VAR(1)



(A) Impact Responses: All Instruments

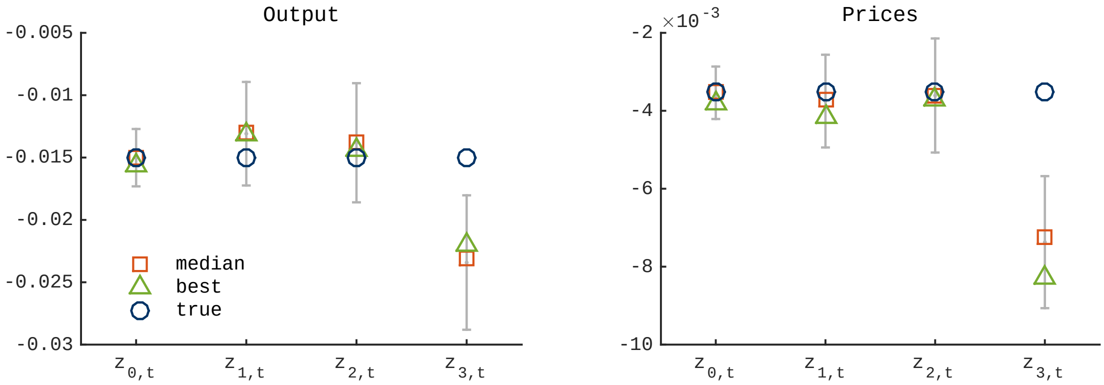
Note: Impact responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(1) in four observables. $z_{0,t}$: observed shock case; $z_{1,t}$: instrument correlates with monetary policy shock only; $z_{2,t}$: instrument also correlates with past spending shocks; $z_{3,t}$ instrument correlates also with past technology shocks. Grey vertical lines are 2 standard deviations error bards from the distribution of impact responses across 5,000 simulated economies of sample size $T = 300$ periods. True impact (blue circle), median across simulations (orange square), minimum distance from median (best) simulation (green triangle).



(B) $z_{1,t}$: external instrument correlates with monetary policy shock only

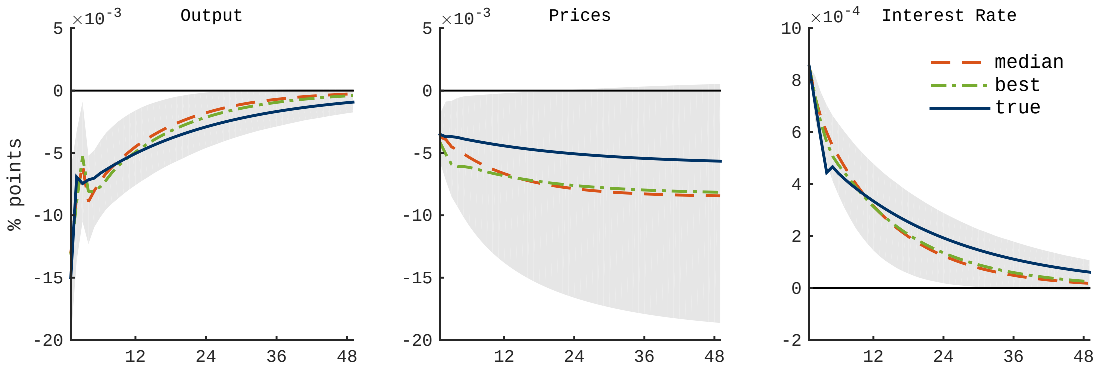
Notes: Impulse responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(1) in four observables. Instrument correlates with monetary policy shocks only. Grey shaded areas denote 90th quantiles of the distribution of IRFs across 5,000 simulated economies of sample size $T = 300$ periods. Model responses (true, blue solid), median across simulations (orange dashed), minimum distance from median (best) simulation (green dash-dotted).

FIGURE B.2: RESPONSES TO MP SHOCK – SIMULATION & VAR(2)



(A) Impact Responses: All Instruments

Note: Impact responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(2) in four observables. $z_{0,t}$: observed shock case; $z_{1,t}$: instrument correlates with monetary policy shock only; $z_{2,t}$: instrument also correlates with past spending shocks; $z_{3,t}$ instrument correlates also with past technology shocks. Grey vertical lines are 2 standard deviations error bards from the distribution of impact responses across 5,000 simulated economies of sample size $T = 300$ periods. True impact (blue circle), median across simulations (orange square), minimum distance from median (best) simulation (green triangle).



(B) $z_{1,t}$: external instrument correlates with monetary policy shock only

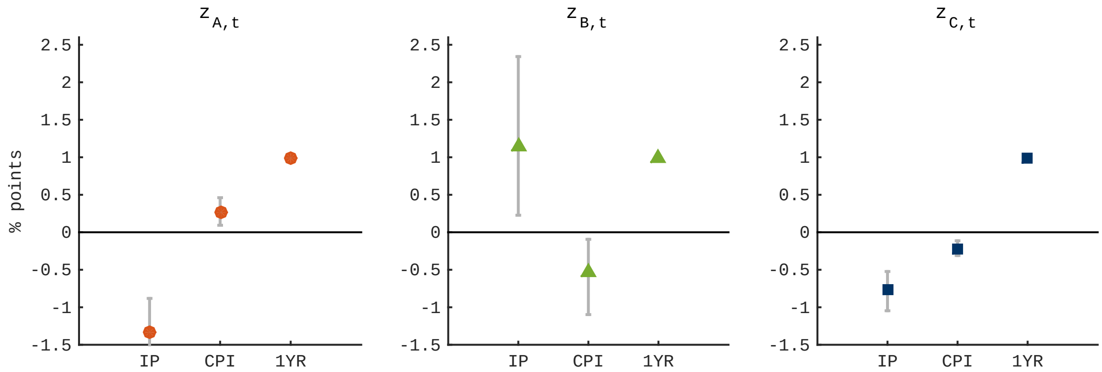
Notes: Impulse responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(2) in four observables. Instrument correlates with monetary policy shocks only. Grey shaded areas denote 90th quantiles of the distribution of IRFs across 5,000 simulated economies of sample size $T = 300$ periods. Model responses (true, blue solid), median across simulations (orange dashed), minimum distance from median (best) simulation (green dash-dotted).

Table B.1: Contamination of Monetary Policy Instruments

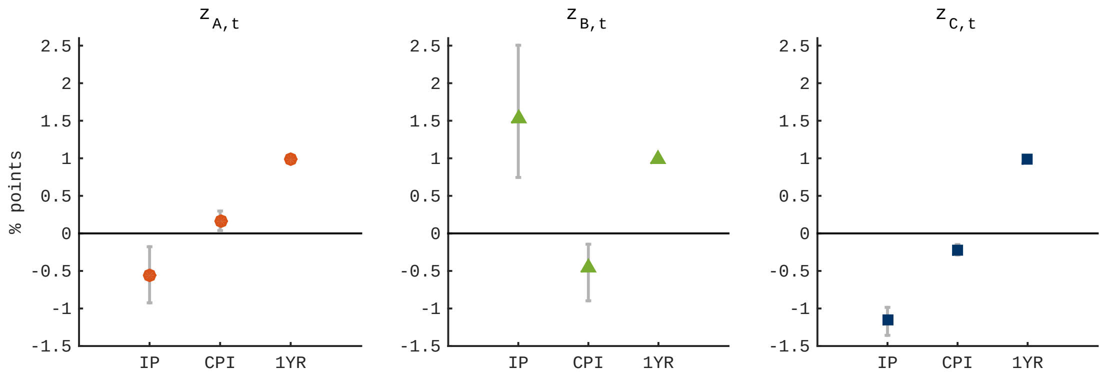
	$z_{\text{A},t}$	$z_{\text{B},t}$	$z_{\text{C},t}$
$f_{1,t-1}$	-0.007**	-0.011***	0.002
	(0.003)	(0.004)	(0.005)
$f_{2,t-1}$	0.000	0.004*	0.001
	(0.002)	(0.002)	(0.002)
$f_{3,t-1}$	0.003	-0.001	0.004
	(0.004)	(0.004)	(0.005)
$f_{4,t-1}$	0.008**	0.008*	0.011*
	(0.004)	(0.004)	(0.006)
$f_{5,t-1}$	-0.005	0.000	0.003
	(0.004)	(0.005)	(0.006)
$f_{6,t-1}$	-0.009***	-0.007***	-0.011**
	(0.003)	(0.003)	(0.005)
$f_{7,t-1}$	-0.009**	-0.006	-0.002
	(0.004)	(0.004)	(0.006)
$f_{8,t-1}$	-0.002	0.001	0.000
	(0.002)	(0.003)	(0.003)
$f_{9,t-1}$	-0.001	-0.002	0.000
	(0.003)	(0.004)	(0.004)
$f_{10,t-1}$	-0.001	0.000	-0.001
	(0.003)	(0.004)	(0.004)
$z_{\text{A},t-1}$	-0.184***		
	(0.052)		
$z_{\text{B},t-1}$		0.204**	
		(0.101)	
$z_{\text{A},t-1}$			-0.009
			(0.075)
constant	-0.006**	-0.011***	-0.000
	(0.003)	(0.003)	(0.004)
R^2	0.097	0.140	0.042
F	2.363	3.616	1.650
F	0.009	0.000	0.087
N	239	238	227

Note: Regressions include a constant and 12 lags of the dependent variable. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$, robust standard errors.

Figure B.3: Impact Responses to Monetary Policy Shocks – 1990:2012



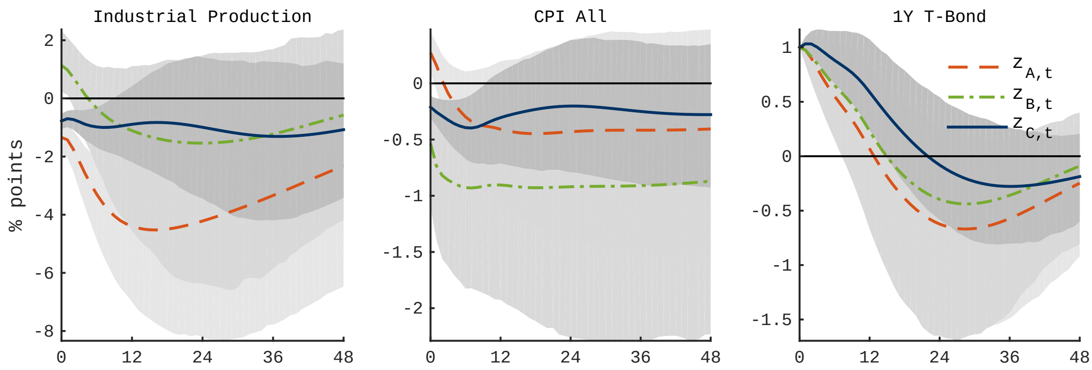
(A) Baseline VAR



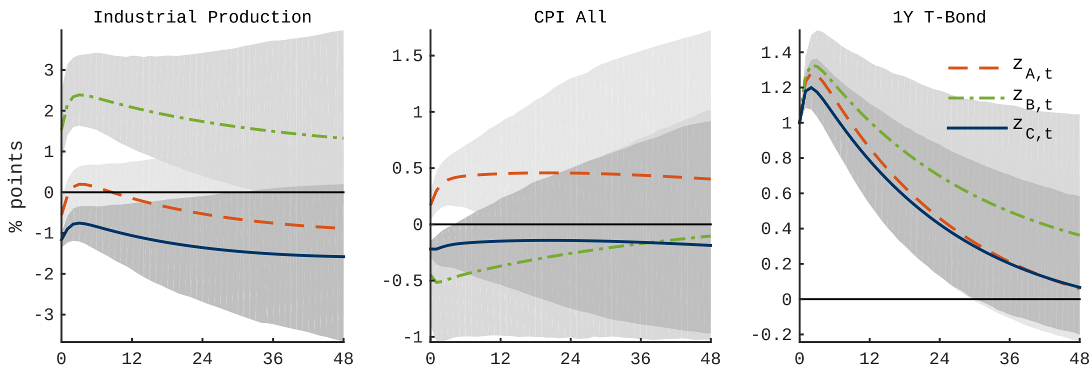
(B) Misspecified VAR

Notes: Baseline VAR(12) in all variables, top panel (A). Misspecified VAR(2) in three variables, bottom panel (B). VARs estimated with standard macroeconomic priors. Identification in all cases uses the full length of the instruments. $z_{\text{A},t}$: high-frequency surprises at scheduled FOMC meetings; $z_{\text{B},t}$: moving average of high-frequency surprises within the month; $z_{\text{C},t}$: residuals of $z_{\text{A},t}$ on Fed Greenbook forecasts. Shaded areas denote 90% posterior coverage bands.

Figure B.4: Responses to Monetary Policy Shocks – 1990:2012



(A) Baseline VAR



(B) Misspecified VAR

Notes: Baseline: VAR(12) in all variables. Misspecified: VAR(2) in three variables. VARs estimated with standard macroeconomic priors. Identification in all cases uses the full length of the instruments. $z_{\text{A},t}$: sum of high-frequency surprises within the month; $z_{\text{B},t}$: moving average of high-frequency surprises within the month; $z_{\text{C},t}$: residuals of $z_{\text{A},t}$ on Fed Greenbook forecasts. Shaded areas denote 90% posterior coverage bands.